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DFG-Graduiertenkolleg
MATHEMATISCHE
KOMPLEXITÄTSREDUKTION

Compact Course Polynomial Optimization – Series 5

https://www.mathcore.ovgu.de/TEACHING/COMPACTCOURSES/2020opt.php

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Exercise 5.1

Get the Matlab toolboxes YALMIP (https://yalmip.github.io) and SeDuMi (http://sedumi.ie.lehigh.edu) running on your notebook. You may also use Octave (https://www.gnu.org/software/octave/) as a free alternative to Matlab.

Test your setup by

a) computing an SOS representation of the polynomial $f = 2 + X_1^2 + X_1^2 X_2^4 - 4X_1X_2$ from Exercise 1.3.

b) confirming part c) of Exercise 1.4 by finding $c \in \mathbb{R}$ such that

$$h(X_1, 1, X_3) + c := X_3^6 - 3X_1^2X_3^2 + X_1^2 + X_1^4 + c$$

is SOS.

Exercise 5.2

Consider the polynomials $f, g_1, g_2, g_3 \in \mathbb{R}[X_1, X_2]$ with

$$f = -X_1^4 - X_2^4 - 2X_1^2X_2^2 + 2X_1^2X_2 + 2X_1X_2^2 + 6X_1^2 - 22X_1X_2 + 6X_2^2 + 6X_1 + 10X_2 - 5$$

and

$$g = (g_1, g_2, g_3) = (X_1 - \frac{1}{2}, X_2 - \frac{1}{2}, 1 - X_1X_2)$$

Compute an algebraic certificate showing that $f \geq 0$ on $\{g \geq 0\}$.

Exercise 5.3

Consider the following integer quadratic programming formulation for the problem of finding a cut of maximum cardinality in an undirected graph $G = (V, E)$:

$$\max \sum_{(i,j) \in E} \frac{1 - x_i x_j}{2} \quad s.t. \;\; x_i \in \{-1, 1\} \quad \forall i \in V. \qquad \text{(MAXCUT)}$$

Formulate the following relaxation as an SDP and test it on several instances of your choice:

$$\max \sum_{(i,j) \in E} \frac{1 - x_i \cdot x_j}{2} \quad s.t. \;\; x_i \in \mathbb{R}^n, \;\; \|x_i\|_2^2 = 1 \quad \forall i \in V,$$

where $n \in \mathbb{N}$.
What is the worst approximation ratio for the optimum of (MAXCUT) you observed?